Chapter 29

6. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.0333\,\mathrm{A})}{4}\left(\frac{1}{0.0572\,\mathrm{m}} + \frac{1}{0.0936\,\mathrm{m}}\right)$$
$$= 2.95 \times 10^{-7}\,\mathrm{T}.$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 29-7(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$, which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2 + b^2) = \frac{\pi(0.0333\text{ A})}{2}[(0.0572\text{ m})^2 + (0.0936\text{ m})^2] = 6.29 \times 10^{-4}\,\mathrm{A\cdot m^2}.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

12. **THINK** We apply the Biot-Savart law to calculate the magnetic field at point P_2. An integral is required since the length of the wire is finite.

EXPRESS We take the x axis to be along the wire with the origin at the right endpoint. The current is in the $+x$ direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from $x = -L$ to $x = 0$.

ANALYZE The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}$$

$$= \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(0.880\ \mathrm{A}\right)}{4\pi\left(0.136\ \mathrm{m}\right)}\frac{0.250\ \mathrm{m}}{\sqrt{(0.250\ \mathrm{m})^2+(0.136\ \mathrm{m})^2}} = 5.68\times10^{-7}\ \mathrm{T}.$$

LEARN In calculating B at P_2, we could have chosen the origin to be at the left endpoint. This only changes the integration limit, but the result remains the same:

$$B = \frac{\mu_0 iR}{4\pi}\int_{0}^{L}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{0}^{L} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}.$$

43. **THINK** The hollow conductor has cylindrical symmetry, so Ampere's law can be applied to calculate the magnetic field due to the current distribution.

EXPRESS Ampere's law states that $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where i_{enc} is the current enclosed by the closed path, or Amperian loop. We choose the Amperian loop to be a circle of radius *r* and concentric with the cylindrical shell. Since the current is uniformly distributed throughout the cross section of the shell, the enclosed current is

$$i_{\text{enc}} = i \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = i\left(\frac{r^2 - b^2}{a^2 - b^2}\right).$$

ANALYZE (a) Thus, in the region $b < r < a$, we have

$$\oint \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_0 i_{\text{enc}} = \mu_0 i\left(\frac{r^2 - b^2}{a^2 - b^2}\right)$$

which gives $B = \dfrac{\mu_0 i}{2\pi\left(a^2 - b^2\right)}\left(\dfrac{r^2 - b^2}{r}\right)$.

(b) At $r = a$, the magnetic field strength is

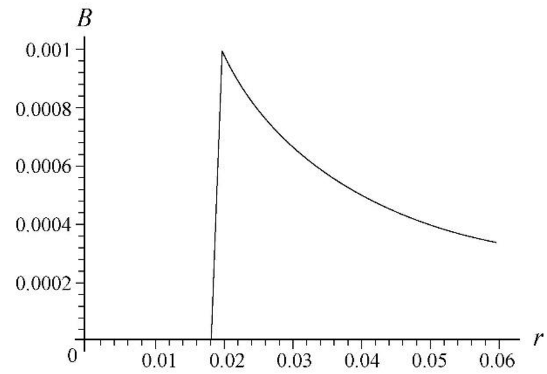
$$\frac{\mu_0 i}{2\pi\left(a^2 - b^2\right)}\left(\frac{a^2 - b^2}{a}\right) = \frac{\mu_0 i}{2\pi a}.$$

At $r = b, B \propto r^2 - b^2 = 0$. Finally, for $b = 0$

$$B = \frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r} = \frac{\mu_0 i r}{2\pi a^2}$$

which agrees with Eq. 29-20.

(c) The field is zero for $r < b$ and is equal to Eq. 29-17 for $r > a$, so this along with the result of part (a) provides a determination of *B* over the full range of values. The graph (with SI units understood) is shown below.



LEARN For $r < b$, the field is zero, and for $r > a$, the field decreases as $1/r$. In the region $b < r < a$, the field increases with r as $r - b^2/r$.

46. The magnitudes of the forces on the sides of the rectangle that are parallel to the long straight wire (with i_1 = 30.0 A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\ \text{sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y}\, dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$\begin{aligned} F &= \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+b}\right) = \frac{\mu_0 i_1 i_2 bL}{2\pi a\left(a+b\right)} \\ &= \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(30.0\ \text{A}\right)\left(20.0\ \text{A}\right)\left(8.00 \times 10^{-2}\ \text{m}\right)\left(30.0 \times 10^{-2}\ \text{m}\right)}{2\pi (2.00 \times 10^{-2}\ \text{m})\left(2.00 \times 10^{-2}\ \text{m} + 8.00 \times 10^{-2}\ \text{m}\right)} = 1.44 \times 10^{-3}\ \text{N}, \end{aligned}$$

and $\vec{F}$ points toward the wire, or $+\hat{\text{j}}$. That is, $\vec{F} = (1.44 \times 10^{-3}\,\text{N})\hat{\text{j}}$ in unit-vector notation.

53. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 29-17,

$$\left|\vec{B}\right| = \frac{\mu_0 i_w}{2\pi r} = 4.8\times10^{-3}\ \text{T}$$

where i_w = 24 A and r = 0.0010 m.

(b) Now the field consists of two contributions (which are anti-parallel) — from the wire (Eq. 29-17) and from a portion of the conductor (Eq. 29-20 modified for annular area):

$$|\vec{B}| = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_0^2 - \pi R_i^2}\right)$$

where r = 0.0030 m, R_i = 0.0020 m, R_o = 0.0040 m, and i_c = 24 A. Thus, we find $|\vec{B}| = 9.3\times10^{-4}\,\text{T}$.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

60. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — “Magnetic field at the center of a circular arc of current,” we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc *H J* contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc *D A* contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(0.593\ \mathrm{A})}{4}\left(\frac{1}{0.0200\ \mathrm{m}} - \frac{1}{0.0780\ \mathrm{m}}\right) = 6.93 \times 10^{-6}\ \mathrm{T}.$$

(b) The direction of the field is into the page.