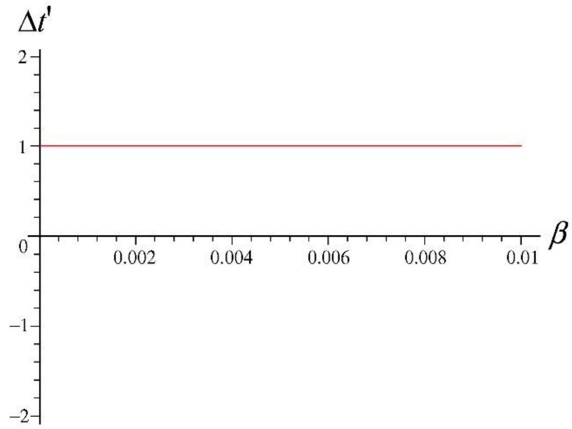
Chapter 37

1. (a) Using Eq. 2′ of Table 37-2, we have

$$\Delta t' = \gamma\left(\Delta t - \frac{v\,\Delta x}{c^2}\right) = \gamma\left(\Delta t - \frac{\beta\,\Delta x}{c}\right) = \gamma\left(1.00\times10^{-6}\,\text{s} - \frac{\beta(400\,\text{m})}{2.998\times10^{8}\,\text{m/s}}\right)$$

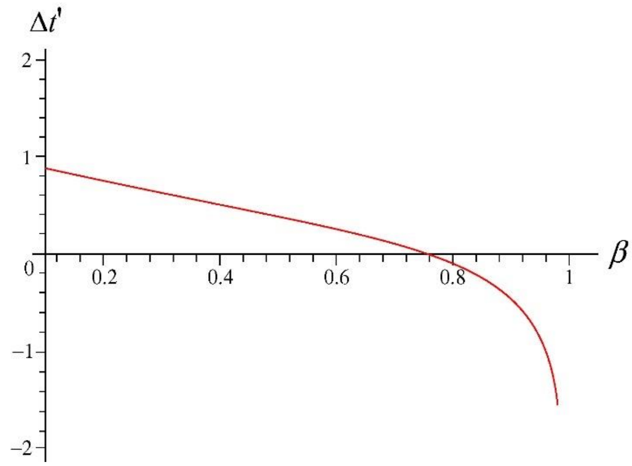
where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ as a function of β in the range $0<\beta<0.01$ is shown below:



Note the limits of the vertical axis are +2 μs and –2 μs. We note how "flat" the curve is in this graph; the reason is that for low values of β, Bullwinkle's measure of the temporal separation between the two events is approximately our measure, namely +1.0 μs. There are no nonintuitive relativistic effects in this case.

(c) A plot of $\Delta t'$ as a function of β in the range $0.1<\beta<1$ is shown below:



(d) Setting

$$\Delta t' = \gamma\left(\Delta t - \frac{\beta\,\Delta x}{c}\right) = \gamma\left(1.00\times10^{-6}\,\text{s} - \frac{\beta(400\,\text{m})}{2.998\times10^{8}\,\text{m/s}}\right) = 0$$

leads to

$$\beta = \frac{c\,\Delta t}{\Delta x} = \frac{(2.998\times10^{8}\,\text{m/s})(1.00\times10^{-6}\,\text{s})}{400\,\text{m}} = 0.7495 \approx 0.750\,.$$

(e) For the graph shown in part (c), as we increase the speed, the temporal separation according to Bullwinkle is positive for the lower values and then goes to zero and finally (as the speed approaches that of light) becomes progressively more negative. For the lower speeds with

$$\Delta t' > 0 \Rightarrow t_A' < t_B' \Rightarrow \quad 0 < \beta < 0.750,$$

according to Bullwinkle event *A* occurs before event *B* just as we observe.

(f) For the higher speeds with

$$\Delta t' < 0 \;\Rightarrow\; \quad t_A' > t_B' \Rightarrow 0.750 < \beta < 1,$$

according to Bullwinkle event *B* occurs before event *A* (the opposite of what we observe).

(g) No, event *A* cannot cause event *B* or vice versa. We note that

$$\Delta x/\Delta t = (400\text{ m})/(1.00\ \mu\text{s}) = 4.00\times10^{8}\text{ m/s} > c.$$

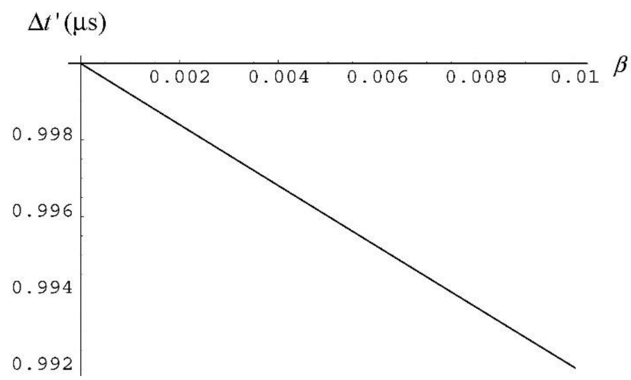
A signal cannot travel from event *A* to event *B* without exceeding *c*, so causal influences cannot originate at *A* and thus affect what happens at *B*, or vice versa.

3. (a) Equation 2′ of Table 37-2 becomes

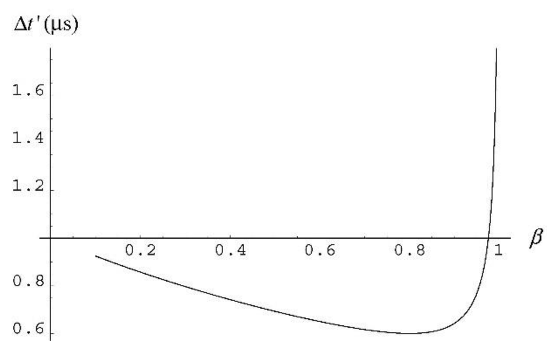
$$\Delta t' = \gamma(\Delta t - \beta\,\Delta x/c) = \gamma\,[1.00\ \mu\text{s} - \beta(240\text{ m})/(2.998\times 10^2\text{ m}/\mu\text{s})]$$
$$= \gamma(1.00 - 0.800\beta)\ \mu\text{s}$$

where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ is shown for the range $0 < \beta < 0.01$:



(c) A plot of $\Delta t'$ is shown for the range $0.1 < \beta < 1$:



(d) The minimum for the $\Delta t'$ curve can be found by taking the derivative and simplifying and then setting equal to zero:

$$\frac{d\Delta t'}{d\beta} = \gamma^3(\beta\Delta t - \Delta x/c) = 0.$$

Thus, the value of β for which the curve is minimum is $\beta = \Delta x/c\Delta t = 240/299.8$, or $\beta = 0.801$.

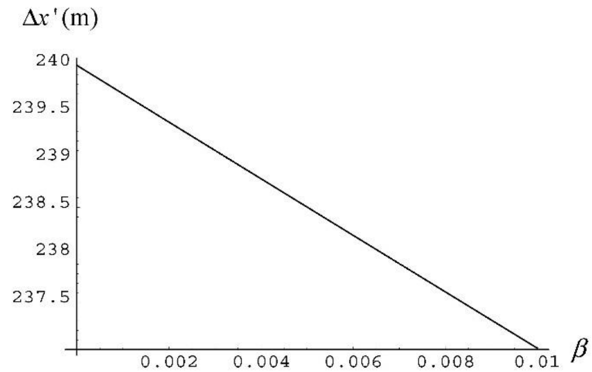
(e) Substituting the value of β from part (d) into the part (a) expression yields the minimum value $\Delta t' = 0.599\ \mu$s.

(f) Yes. We note that $\Delta x/\Delta t = 2.4\times10^{8}$ m/s < c. A signal can indeed travel from event A to event B without exceeding c, so causal influences can originate at A and thus affect what happens at B. Such events are often described as being "time-like separated" – and we see in this problem that it is (always) possible in such a situation for us to find a frame of reference (here with $\beta \approx 0.801$) where the two events will seem to be at the same location (though at different times).

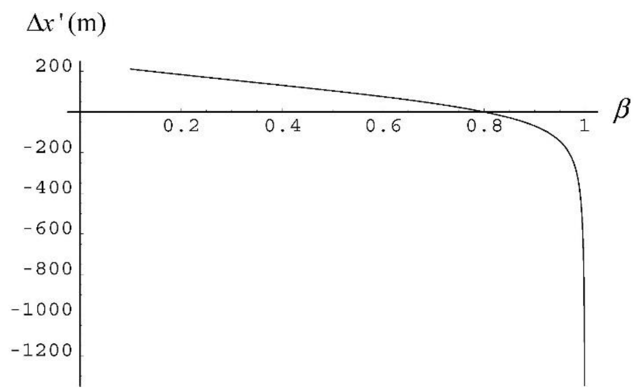
4. (a) Equation 1′ of Table 37-2 becomes

$$\Delta x' = \gamma(\Delta x - \beta c\,\Delta t) = \gamma[(240\text{ m}) - \beta(299.8\text{ m})]\,.$$

(b) A plot of $\Delta x'$ for $0 < \beta < 0.01$ is shown below:



(c) A plot of $\Delta x'$ for $0.1 < \beta < 1$ is shown below:



We see that $\Delta x'$ decreases from its $\beta = 0$ value (where it is equal to $\Delta x = 240$ m) to its zero value (at $\beta \approx 0.8$), and continues (without bound) downward in the graph (where it is negative, implying event B has a *smaller* value of x' than event A!).

(d) The zero value for $\Delta x'$ is easily seen (from the expression in part (b)) to come from the condition $\Delta x - \beta c\,\Delta t = 0$. Thus $\beta = 0.801$ provides the zero value of $\Delta x'$.

24. We wish to adjust Δt so that

$$0 = \Delta x' = \gamma\left(\Delta x - v\ \Delta t\right) = \gamma(-520\,\text{m} - v\ \Delta t)$$

in the limiting case of $|v| \to c$. Thus,

$$\Delta t = \frac{\Delta x}{v} = \frac{\Delta x}{c} = \frac{520\,\text{m}}{2.998\times10^{8}\ \text{m/s}} = 1.73\times10^{-6}\ \text{s}\ .$$

42. From the value of L in the graph when $\beta = 0$, we infer that L_0 in Eq. 37-13 is 0.80 m. Thus, that equation (which describes the curve in Fig. 37-23) with SI units understood becomes

$$L = L_0\sqrt{1-(v/c)^2} = (1.60\,\mathrm{m})\sqrt{1-\beta^2}\ .$$

If we set $\beta = 0.98$ in this expression, we obtain approximately 0.32 m for L.

44. The line in the graph is described by Eq. 1 in Table 37-2:

$$\Delta x = v\gamma\,\Delta t' + \gamma\,\Delta x' = (\text{“slope”})\Delta t' \;+\; \text{“}y\text{ intercept”}$$

where the “slope” is 7.0×10^8 m/s. Setting this value equal to $v\gamma$ leads to $v = 2.8 \times 10^8$ m/s and $\gamma = 2.54$. Since the “y intercept” is 2.0 m, we see that dividing this by γ leads to $\Delta x' = 0.79$ m.