Chapter 27

28. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let *i* be the current in either battery and take it to be positive to the left. According to the junction rule the current in *R* is 2*i* and it is positive to the right. The loop rule applied to either loop containing a battery and *R* yields

$$\mathcal{E} - ir - 2iR = 0 \quad \Rightarrow \quad i = \frac{\mathcal{E}}{r + 2R}.$$

The power dissipated in *R* is

$$P = (2i)^2 R = \frac{4\mathcal{E}^2 R}{\left(r + 2R\right)^2}.$$

We find the maximum by setting the derivative with respect to *R* equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\mathcal{E}^2}{(r+2R)^3} - \frac{16\mathcal{E}^2 R}{(r+2R)^3} = \frac{4\mathcal{E}^2 (r-2R)}{(r+2R)^3}.$$

The derivative vanishes (and *P* is a maximum) if $R = r/2$. With $r = 0.400\ \Omega$, we have $R = 0.200\ \Omega$.

(b) We substitute $R = r/2$ into $P = 4\mathcal{E}^2 R/(r + 2R)^2$ to obtain

$$P_{\max} = \frac{4\mathcal{E}^2 (r/2)}{[r + 2(r/2)]^2} = \frac{\mathcal{E}^2}{2r} = \frac{(14.0\ \text{V})^2}{2(0.400\ \Omega)} = 245\ \text{W}.$$

29. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\mathcal{E} = ir + (2i)R$ with $r = 0.400\ \Omega$ and $R = 2.00r$, we get

$$i_R = 2i = \frac{2\mathcal{E}}{r+2R} = \frac{2(12.0\text{V})}{0.400\ \Omega + 2(0.800\ \Omega)} = 12.0\ \text{A}.$$

(b) When connected in series $2\mathcal{E} - i_R r - i_R r - i_R R = 0$, or $i_R = 2\mathcal{E}/(2r + R)$. The result is

$$i_R = 2i = \frac{2\mathcal{E}}{2r+R} = \frac{2(12.0\ \text{V})}{2(0.400\ \Omega) + 0.800\ \Omega} = 15.0\ \text{A}.$$

(c) They are in series arrangement, since $R > r$.

(d) If $R = r/2.00$, then for parallel connection,

$$i_R = 2i = \frac{2\mathcal{E}}{r+2R} = \frac{2(12.0\text{V})}{0.400\ \Omega + 2(0.200\ \Omega)} = 30.0\ \text{A}.$$

(e) For series connection, we have

$$i_R = 2i = \frac{2\mathcal{E}}{2r+R} = \frac{2(12.0\ \text{V})}{2(0.400\ \Omega) + 0.200\ \Omega} = 24.0\ \text{A}.$$

(f) They are in parallel arrangement, since $R < r$.

30. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\mathcal{E}_2 = \mathcal{E}_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through $\mathcal{E}_2$ and $\mathcal{E}_3$ are the same: $i_2 = i_3 = i$. Therefore, the current through $\mathcal{E}_1$ is $i_1 = 2i$. Then from $V_b - V_a = \mathcal{E}_2 - iR_2 = \mathcal{E}_1 + (2R_1)(2i)$ we get

$$i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{4R_1 + R_2} = \frac{5.0\,\text{V} - 3.0\,\text{V}}{4(2.0\,\Omega) + 4.0\,\Omega} = 0.32\,\text{A}.$$

Therefore, the current through $\mathcal{E}_1$ is $i_1 = 2i = 0.64$ A.

(b) The direction of i_1 is downward.

(c) The current through $\mathcal{E}_2$ is $i_2 = 0.14$ A.

(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.18$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \mathcal{E}_2 = -(0.32\text{ A})(4.0\ \Omega) + 4.0\text{ V} = 2.7\text{ V}$.

35. The current in the circuit is

$$i = (150\text{ V} - 80\text{ V})/(4.0\ \Omega + 6.0\ \Omega) = 7.0\text{ A}.$$

So from $V_Q + 150\text{ V} - (6.0\ \Omega)i = V_P$, we get

$$V_Q = 100\text{ V} + (6.0\ \Omega)(7.0\text{ A}) - 150\text{ V} = -8.0\text{ V}.$$

39. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2 \frac{\varepsilon}{R_1 + R_2} = (15.0\ \text{k}\Omega)\left(\frac{20.0\text{V}}{10.0\ \text{k}\Omega + 15.0\ \text{k}\Omega}\right) = 12.0\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.01400 s, we obtain

$$V = (12)e^{-0.014/(15000)(0.4\times10^{-6})} = 1.163\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 1.163/15000 = 7.76×10^{-5} A.

48. First, we note in V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (2.50\ \text{A})(8.00\ \Omega + 4.00\ \Omega) = 30.0\ \text{V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4 = (30.0\ \text{V})/(16.0\ \Omega) = 1.88\ \text{A}$.

By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 1.88\ \text{A} + 2.50\ \text{A} = 4.38\ \text{A},$$

so its voltage is $V_2 = (2.00\ \Omega)(4.38\ \text{A}) = 8.75\ \text{V}$.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 38.8\ \text{V}$ (implying that the current through it is $i_3 = V_3/(2.00\ \Omega) = 19.38\ \text{A}$).

The junction rule now gives the current in R_1 as

$$i_1 = i_2 + i_3 = 4.38\ \text{A} + 19.38\ \text{A} = 23.8\ \text{A},$$

implying that the voltage across it is $V_1 = (23.8\ \text{A})(2.00\ \Omega) = 47.5\ \text{V}$. Therefore, by the loop rule,

$$\mathcal{E} = V_1 + V_3 = 47.5\ \text{V} + 38.8\ \text{V} = 86.3\ \text{V}.$$