Chapter 36

38. (a) Employing Eq. 36-3 with the small angle approximation (sin $\theta \approx$ tan $\theta = y/D$ where y locates the minimum relative to the middle of the pattern), we find (with m = 1)

$$D = \frac{ya}{m\lambda} = \frac{(0.90\text{ mm})(0.40\text{ mm})}{4.50\times10^{-4}\text{ mm}} = 800\text{ mm} = 80\text{ cm}$$

which places the screen 80 cm away from the slit.

(b) The above equation gives for the value of y (for m = 3)

$$y = \frac{(3)\lambda D}{a} = \frac{(3)(4.50\times10^{-4}\text{ mm})(800\text{ mm})}{(0.40\text{ mm})} = 2.7\,\text{mm} .$$

Subtracting this from the first minimum position y = 0.90 mm, we find the result $\Delta y = 1.8\text{ mm}$.

41. Consider two of the rays shown in Fig. 36-49, one just above the other. The extra distance traveled by the lower one may be found by drawing perpendiculars from where the top ray changes direction (point P) to the incident and diffracted paths of the lower one. Where these perpendiculars intersect the lower ray's paths are here referred to as points A and C. Where the bottom ray changes direction is point B. We note that angle $\angle APB$ is the same as ψ, and angle BPC is the same as θ (see Fig. 36-49). The difference in path lengths between the two adjacent light rays is

$$\Delta x = |AB| + |BC| = d \sin \psi + d \sin \theta.$$

The condition for bright fringes to occur is therefore

$$\Delta x = d(\sin\psi + \sin\theta) = m\lambda$$

where $m = 0, 1, 2, \ldots$. If we set $\psi = 0$ then this reduces to Eq. 36-25.

45. One strategy is to divide Eq. 36-25 by Eq. 36-3, assuming the same angle (a point we'll come back to, later) and the same light wavelength for both:

$$\frac{m}{m'} = \frac{m\lambda}{m'\lambda} = \frac{d\sin\theta}{a\sin\theta} = \frac{d}{a}\,.$$

We recall that d is measured from middle of transparent strip to the middle of the next transparent strip, which in this particular setup means $d = 2a$. Thus, $m/m' = 2$, or $m = 2m'$.

Now we interpret our result. First, the division of the equations is not valid when $m = 0$ (which corresponds to $\theta = 0$), so our remarks do not apply to the $m = 0$ maximum. Second, Eq. 36-25 gives the "bright" interference results, and Eq. 36-3 gives the "dark" diffraction results (where the latter overrules the former in places where they coincide – see Figure 36-17 in the textbook). For m' = any nonzero integer, the relation $m = 2m'$ implies that m = any nonzero *even* integer. As mentioned above, these are occurring at the same angle, so the even integer interference maxima are eliminated by the diffraction minima.

50. The condition for a minimum of a single-slit diffraction pattern is

$$a\sin\theta = m\lambda$$

where *a* is the slit width, λ is the wavelength, and *m* is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is 1.20° for *m* = 1. Thus,

$$a = \frac{m\lambda}{\sin\theta} = \frac{415\times10^{-9}\text{ m}}{\sin 1.20^\circ} = 1.98\times10^{-5}\text{ m}\ .$$

51. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda / a)$. The maxima in the double-slit pattern are located at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$, we find $-6 < m < +6$, or, since m is an integer, $-5 \le m \le +5$. Thus, we find eleven values of m that satisfy this requirement.