Chapter 26

6. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

$$J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L / A}} = \sqrt{\frac{P}{\rho L A}} = \sqrt{\frac{1.0\,\mathrm{W}}{\pi\left(3.5\times10^{-5}\ \Omega\cdot\mathrm{m}\right)\left(2.0\times10^{-2}\ \mathrm{m}\right)\left(5.0\times10^{-3}\ \mathrm{m}\right)^2}}$$

$$= 1.3\times10^{5}\ \mathrm{A/m^2}.$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\,\mathrm{W}}{\pi\left(5.0\times10^{-3}\ \mathrm{m}\right)^2\left(1.3\times10^{5}\ \mathrm{A/m^2}\right)} = 9.4\times10^{-2}\ \mathrm{V}.$$

12. (a) Referring to Fig. 26-33, the electric field would point down (toward the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction electrons would be "drifting" upward in the strip.

(b) Equation 24-6 immediately gives 18 eV, or (using $e = 1.60 \times 10^{-19}$ C) 2.88×10^{-18} J for the work done by the field (which equals, in magnitude, the potential energy change of the electron).

(c) Since the electrons don't (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 18 eV or 2.88×10^{-18} J.

13. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}|\, dA = \int_0^R kr^2 2\pi r\; dr = \frac{1}{2} k\pi R^4 = 1.69 \text{ A}$$

where $k = 2.75 \times 10^{10}$ A/m^4 and $R = 0.00250$ m. The rate of thermal energy generation is found from Eq. 26-26: $P = iV = 101$ W. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t =$ (101 J/s)(3600 s) = 3.64×10^5 J.

29. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3}\ \text{A})(1800\ \Omega) = 90\ \text{V}$.

35. (a) Let ΔT be the change in temperature and κ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L\, \Delta T$ and

$$\frac{\Delta L}{L} = \kappa\ \Delta T = (1.7\times10^{-5}\ /\ \text{K})(1.0°\text{C}) = 1.7\times10^{-5}.$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of κ used in this calculation is not inconsistent with the other units involved.

(b) Incorporating a factor of 2 for the two-dimensional nature of A, the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa\ \Delta T = 2(1.7\times10^{-5}\ /\ \text{K})(1.0\ \text{C}°) = 3.4\times10^{-5}$$

which is 0.0034%.

(c) For small changes in the resistivity ρ, length L, and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho}\Delta\rho + \frac{\partial R}{\partial L}\Delta L + \frac{\partial R}{\partial A}\Delta A.$$

Since $R = \rho L/A$, $\partial R/\partial\rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta\rho/\rho = \alpha\,\Delta T$, where α is the temperature coefficient of resistivity for copper (4.3×10^{-3}/K $= 4.3 \times 10^{-3}$/C°, according to Table 27-1). Thus,

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} = (\alpha + \kappa - 2\kappa)\Delta T = (\alpha - \kappa)\Delta T$$
$$= (4.3\times10^{-3}\ /\ \text{C}° - 1.7\times10^{-5}\ /\ \text{C}°)(1.0\ \text{C}°) = 4.3\times10^{-3}.$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta\rho/\rho$ term in the above calculation.

(d) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

54. The number density of conduction electrons in copper is $n = 8.49 \times 10^{28}$ /m^3. The electric field in 'section 2' is (15.0 μV)/(2.00 m) = 7.50 μV/m. Since $\rho = 1.69 \times 10^{-8}\ \Omega \cdot$ m for copper (see Table 26-1) then Eq. 26-10 leads to a current density vector of magnitude

$$J_2 = (7.50\ \mu\text{V/m})/(1.69 \times 10^{-8}\ \Omega \cdot \text{m}) = 443.7\ \text{A/m}^2$$

in section 2. Conservation of electric current from section 1 into section 2 implies

$$J_1 A_1 = J_2 A_2 \quad \Rightarrow \quad J_1(4\pi R^2) = J_2(\pi R^2)$$

(see Eq. 26-5). This leads to J_1 = 110.9 A/m^2. Now, for the drift speed of conduction-electrons in section 1, Eq. 26-7 immediately yields

$$v_d = \frac{J_1}{ne} = 8.17 \times 10^{-9}\ \text{m/s}\ .$$