Chapter 25

10. Let

$$C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A\kappa_1/4d,$$
$$C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A\kappa_2/2d,$$
$$C_3 = \varepsilon_0 A\kappa_3/2d.$$

Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus,

$$C = C_1 + \frac{C_2C_3}{C_2+C_3} = \frac{\varepsilon_0 A\kappa_1}{4d} + \frac{(\varepsilon_0 A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2+\kappa_3/2} = \frac{\varepsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right).$$

With $A = 2.53\times10^{-3}\,\mathrm{m}^2$, $d = 3.56\times10^{-3}\,\mathrm{m}$, $\kappa_1 = 21.0$, $\kappa_2 = 42.0$ and $\kappa_3 = 58.0$, we find the capacitance to be

$$C = \frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(2.53\times10^{-3}\ \mathrm{m^2})}{4(3.56\times10^{-3}\ \mathrm{m})}\left(21.0 + \frac{2(42.0)(58.0)}{42.0+58.0}\right) = 1.10\times10^{-10}\,\mathrm{F}.$$

13. Initially the capacitors C_1, C_2, and C_3 form a series combination equivalent to a single capacitor, which we denote C_{123}. Solving the equation

$$\frac{1}{C_{123}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}=\frac{C_1C_2+C_2C_3+C_1C_3}{C_1C_2C_3},$$

we obtain $C_{123} = 2.40\ \mu$F. With $V = 12.0$ V, we then obtain $q = C_{123}V = 28.8\ \mu$C. In the final situation, C_2 and C_4 are in parallel and are thus effectively equivalent to $C_{24} = 12.0\ \mu\text{F}$. Similar to the previous computation, we use

$$\frac{1}{C_{1234}}=\frac{1}{C_1}+\frac{1}{C_{24}}+\frac{1}{C_3}=\frac{C_1C_{24}+C_{24}C_3+C_1C_3}{C_1C_{24}C_3}$$

and find $C_{1234} = 3.00\ \mu$F. Therefore, the final charge is $q = C_{1234}V = 36.0\ \mu$C.

(a) This represents a change (relative to the initial charge) of $\Delta q = 7.20\ \mu$C.

(b) The capacitor C_{24} which we imagined to replace the parallel pair C_2 and C_4, is in series with C_1 and C_3 and thus also has the final charge $q = 36.0\ \mu$C found above. The voltage across C_{24} would be

$$V_{24}=\frac{q}{C_{24}}=\frac{36.0\ \mu\text{C}}{12.0\ \mu\text{F}}=3.00\text{ V}.$$

This is the same voltage across each of the parallel pairs. In particular, $V_4 = 3.00$ V implies that $q_4 = C_4 V_4 = 18.0\ \mu$C.

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

24. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by (see Eq. 25-12)

$$E = \frac{q}{2\pi\varepsilon_0 L r},$$

and the energy density at that point is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2}.$$

The corresponding energy in the cylinder is the volume integral $U_R = \int u\, d\mathbb{V}$. Now, $d\mathbb{V} = 2\pi r L\, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} 2\pi r L\, dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{R}{a}\right).$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right).$$

We want the ratio U_R/U_b to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln\left(\sqrt{b/a}\right)$, $\ln(R/a) = \ln\left(\sqrt{b/a}\right)$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

33. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 9.00\,\mu\text{F} + \frac{(15.0\,\mu\text{F})(5.00\,\mu\text{F})}{15.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 12.8\,\mu\text{F}.$$

36. (a) The charge q_3 in the figure is $q_3 = C_3V = (4.00\,\mu\text{F})(250\text{ V}) = 1.00\text{ mC}$.

(b) $V_3 = V = 250$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 0.125\text{ J}$.

(d) From the figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1 + C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(250\text{ V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 0.833\text{ mC}.$$

(e) $V_1 = q_1/C_1 = 8.33 \times 10^{-4}$ C/10.0 μF = 83.3 V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 34.7\text{ mJ}$.

(g) From part (d), we have $q_2 = q_1 = 0.833\text{ mC}$.

(h) $V_2 = V - V_1 = 250\text{ V} - 83.3\text{ V} = 167$ V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 69.4\text{ mJ}$.

51. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know C_1 and C_2. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.57 \times 10^{-11}\ \text{F} ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 1.16 \times 10^{-10}\ \text{F} .$$

This leads to

$$q_1 = C_1V_1 = 1.38 \times 10^{-9}\ \text{C},\ \ q_2 = C_2V_2 = 3.07 \times 10^{-10}\ \text{C}.$$

The addition of these gives the desired result: $q_{tot} = 1.69 \times 10^{-9}$ C. Alternatively, the circuit could be reduced to find the q_{tot}.