Chapter 24

7. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r=\sqrt{z^2+R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}}=\frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq$$
$$=\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E=-\frac{\partial V}{\partial z}=-\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z)$$
$$=\frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

17. (a) The work done by the electric field is

$$W=\int_i^f q_0\vec{E}\cdot d\vec{s}=\frac{q_0\sigma}{2\varepsilon_0}\int_0^d dz=\frac{q_0\sigma d}{2\varepsilon_0}=\frac{(1.60\times10^{-19}\,\text{C})(3.84\times10^{-12}\ \text{C/m}^2)(0.00173\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}$$
$$=6.01\times10^{-23}\,\text{J}.$$

(b) Since

$$V-V_0=-W/q_0=-\sigma z/2\varepsilon_0,$$

with V_0 set to be zero on the sheet, the electric potential at P is

$$V=-\frac{\sigma z}{2\varepsilon_0}=-\frac{(3.84\times10^{-12}\ \text{C/m}^2)(0.00173\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}=-3.75\times10^{-4}\ \text{V}.$$

25. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 24-18). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=3.50} \vec{E} \cdot d\vec{s} = -2.25\int_0^{3.50} x\,dx = -2.25\left(\frac{3.50^2}{2}\right)$$

which yields $V_B - V_A = -13.8$ V.

34. (a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)\,dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}\,dx.$$

Thus,

$$\begin{aligned} V &= \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}\,dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right) \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.6\times10^{-12}\,\text{C/m}^2)\left(\sqrt{(0.0850\ \text{m})^2+(0.0356\ \text{m})^2}-0.0356\ \text{m}\right) \\ &= 1.66\times10^{-2}\ \text{V}. \end{aligned}$$

(b) The y component of the field there is

$$\begin{aligned} E_y &= -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right) \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.6\times10^{-12}\,\text{C/m}^2)\left(1-\frac{0.0356\ \text{m}}{\sqrt{(0.0850\ \text{m})^2+(0.0356\ \text{m})^2}}\right) \\ &= 0.180\ \text{N/C}. \end{aligned}$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at (x, y). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

46. (a) The electric field between the plates is leftward in Fig, 24-59 since it points toward lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2} m_p v_0^2 + qV_1 = \frac{1}{2} m_p v^2 + qV_2 \ .$$

Using $q = +1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, $v_0 = 120 \times 10^3$ m/s, $V_1 = -70$ V, and $V_2 = -50$ V, we obtain the final speed $v = 1.03 \times 10^5$ m/s. We note that the value of d is not used in the solution.

59. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{2d} + \frac{q_2 Q}{d}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{2d} + \frac{(-q_1/2)Q}{d}\right) = 0.$$