Chapter 23

1. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\jmath} = (0.850\text{ m})^2\hat{\jmath}$.

(a) $\Phi = \left(6.00\ \frac{N}{C}\right)\hat{\imath} \cdot (0.850\text{ m})^2\hat{\jmath} = 0.$

(b) $\Phi = \left(-2.00\ \frac{N}{C}\right)\hat{\jmath} \cdot (0.850\text{ m})^2\hat{\jmath} = \frac{-1.45\text{ N}\cdot m^2}{C}.$

(c) $\Phi = \left[\left(-3.00\ \frac{N}{C}\right)\hat{\imath} + \left(400\frac{N}{C}\right)\hat{k}\right] \cdot (0.850\text{ m})^2\hat{\jmath} = 0.$

(d) The total flux of a uniform field through a closed surface is always zero.

8. Equation 23-6 (Gauss' law) gives $\varepsilon_0\Phi = q_{\text{enc}}$.

(a) Thus, the value $\Phi = 4.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$ for small r leads to

$$q_{\text{central}} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}) = 3.54\times10^{-6}\,\text{C} \approx 3.5\times10^{-6}\,\text{C}\ .$$

(b) The next value that Φ takes is $\Phi = -8.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$, which implies that $q_{\text{enc}} = -7.00\times10^{-6}\,\text{C}$. But we have already accounted for some of that charge in part (a), so the result for part (b) is

$$q_A = q_{\text{enc}} - q_{\text{central}} = -10.54\times10^{-6}\,\text{C} \approx -11\ \mu\text{C}.$$

(c) Finally, the large r value for Φ is $\Phi = 1.2\times10^6\ \text{N}\cdot\text{m}^2/\text{C}$, which implies that $q_{\text{total enc}} = 10.6\times10^{-6}\ \text{C}$. Considering what we have already found, then the result is $q_{\text{total enc}} - q_A - q_{\text{central}} = +18\ \mu C$.

9. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

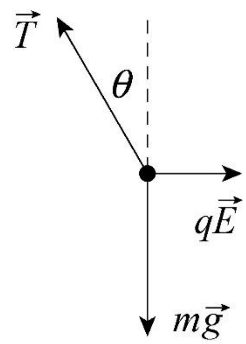
$$\sigma = \frac{q}{\pi D^2} = \frac{3.3\times10^{-6}\ \text{C}}{\pi\left(0.85\ \text{m}\right)^2} = 1.5\times10^{-6}\ \text{C/m}^2.$$

(b) Equation 23-11 gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1.5\times10^{-6}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.6\times10^5\ \text{N/C}.$$

26. **THINK** Since the non-conducting charged ball is in equilibrium with the non-conducting charged sheet (see Fig. 23-49), both the vertical and horizontal components of the net force on the ball must be zero.

EXPRESS The forces acting on the ball are shown in the diagram below.



The gravitational force has magnitude *mg*, where *m* is the mass of the ball; the electrical force has magnitude *qE*, where *q* is the charge on the ball and *E* is the magnitude of the electric field at the position of the ball; and the tension in the thread is denoted by *T*. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical. Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields

$$qE - T \sin \theta = 0$$

and the sum of the vertical components yields

$$T \cos\theta - mg = 0 .$$

We solve for the electric field *E* and deduce σ, the charge density of the sheet, from $E = \sigma/2\varepsilon_0$ (see Eq. 23-13).

ANALYZE The expression $T = qE/\sin \theta$, from the first equation, is substituted into the second to obtain $qE = mg \tan \theta$. The electric field produced by a large uniform sheet of charge is given by $E = \sigma/2\varepsilon_0$, so

$$\frac{q\sigma}{2\varepsilon_0} = mg \tan\theta$$

and we have

$$\sigma = \frac{2\varepsilon_0 mg \tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(5.3\times10^{-6}\ \mathrm{kg}\right)\left(9.8\ \mathrm{m/s^2}\right)\tan 30^\circ}{2.0\times10^{-8}\ \mathrm{C}}$$
$$= 2.7\times10^{-8}\ \mathrm{C/m^2}.$$

LEARN Since both the sheet and the ball are positively charged, the force between them is repulsive. This is balanced by the horizontal component of the tension in the thread. The angle the thread makes with the vertical direction increases with the charge density of the sheet.

28. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r = 0$, the above equation implies $E = 0$.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(8.00\times10^{-15}\,\mathrm{C})}{2(2.00\times10^{-2}\,\mathrm{m})^2} = 8.99\times10^{-2}\ \mathrm{N/C}.$$

(c) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(8.00\times10^{-15}\,\mathrm{C})}{(2.00\times10^{-2}\,\mathrm{m})^2} = 0.180\ \mathrm{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(8.00\times10^{-15}\,\mathrm{C})}{(1.50\times2.00\times10^{-2}\,\mathrm{m})^2} = 0.0799\ \mathrm{N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0.$ Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E} \cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -8.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.$$

59. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{2\lambda_1}{4\pi\varepsilon_0 (x + L/2)} + \frac{2\lambda_2}{4\pi\varepsilon_0 (x - L/2)} \ .$$

Setting this equal to zero and solving for x we find

$$x = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)\frac{L}{2} = \left(\frac{8.0\ \mu\text{C/m} - (-2.0\ \mu\text{C/m})}{8.0\ \mu\text{C/m} + (-2.0\ \mu\text{C/m})}\right)\frac{8.0\ \text{cm}}{2} = 6.7\ \text{cm} .$$