Chapter 22

17. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22.03 — "Electric field of a charged circular rod," illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\rm arc} = \frac{\lambda}{4\pi\varepsilon_0 r} \left[\sin(\theta/2) - \sin(-\theta/2) \right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with θ expressed in radians. Thus, using R instead of r, we obtain

$$E_{\rm arc} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} \ .$$

Thus, the problem requires $E_{arc} = \frac{1}{2} E_{particle}$, where $E_{particle}$ is given by Eq. 22-3. Hence,

$$\frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2 \theta} = \frac{1}{2} \frac{Q}{4\pi\varepsilon_0 R^2} \implies \sin\frac{\theta}{2} = \frac{\theta}{4}$$

where we note, again, that the angle is in radians. The approximate solution to this equation is θ = 3.791 rad \approx 217°.

20. From symmetry, we see that the net field at *P* is twice the field caused by the upper semicircular charge $+q = \lambda(\pi R)$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E}_{\rm net} = 2\left(-\vec{j}\right) \frac{\lambda}{4\pi\varepsilon_0 R} \sin\theta \bigg|_{-90^\circ}^{90^\circ} = -\left(\frac{q}{\varepsilon_0 \pi^2 R^2}\right) j.$$

(a) With $R = 4.23 \times 10^{-2}$ m and $q = 3.90 \times 10^{-11}$ C, $|\vec{E}_{net}| = 250$ N/C.

(b) The net electric field \vec{E}_{net} points in the $-\hat{j}$ direction, or -90° counterclockwise from the +x axis.

21. We assume q > 0. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin \theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

(a) Using Eq. 22-3 (with the 2 and sin θ factors just discussed) the magnitude is

$$\begin{aligned} \left|\vec{E}\right| &= 2\int_0^{L/2} \left(\frac{dq}{4\pi\varepsilon_0 r^2}\right) \sin\theta = \frac{2}{4\pi\varepsilon_0} \int_0^{L/2} \left(\frac{\lambda \ dx}{x^2 + R^2}\right) \left(\frac{y}{\sqrt{x^2 + R^2}}\right) \\ &= \frac{\lambda R}{2\pi\varepsilon_0} \int_0^{L/2} \frac{dx}{\left(x^2 + R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0} \cdot \frac{x}{R^2\sqrt{x^2 + R^2}} \bigg|_0^{L/2} \\ &= \frac{q}{2\pi\varepsilon_0 LR} \frac{L/2}{\sqrt{\left(L/2\right)^2 + R^2}} = \frac{q}{2\pi\varepsilon_0 R} \frac{1}{\sqrt{L^2 + 4R^2}} \end{aligned}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 9.56 \times 10^{-12}$ C, L = 0.0850 m, and R = 0.0600 m, we have $|\vec{E}| = 19.5$ N/C.

(b) As noted above, the electric field \vec{E} points in the +y direction, or +90° counterclockwise from the +x axis.

29. The smallest arc is of length $L_1 = \pi r_1/2 = \pi R/2$; the middle-sized arc has length $L_2 = \pi r_2/2 = \pi (2R)/2 = \pi R$; and, the largest arc has $L_3 = \pi (3R)/2$. The charge per unit length for each arc is $\lambda = q/L$ where each charge q is specified in the figure. Thus, we find the net electric field to be

$$E_{\rm net} = \frac{\lambda_1 (2\sin 45^\circ)}{4\pi\varepsilon_0 r_1} + \frac{\lambda_2 (2\sin 45^\circ)}{4\pi\varepsilon_0 r_2} + \frac{\lambda_3 (2\sin 45^\circ)}{4\pi\varepsilon_0 r_3} = \frac{Q}{\sqrt{2}\pi^2\varepsilon_0 R^2}$$

which yields $E_{\text{net}} = 1.46 \times 10^7 \text{ N/C}$.

(b) The direction is -45° , measured counterclockwise from the +x axis.

32. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$U = -\vec{p} \cdot \vec{E} = -100 \times 10^{-28} \,\mathrm{J}.$$

If E = 40 N/C, we find $p = 2.5 \times 10^{-28}$ C·m.

33. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where *E* is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time *t* is $x = \frac{1}{2}a_pt^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_et^2$. They pass each other when their coordinates are the same, or

$$\frac{1}{2}a_{p}t^{2} = L + \frac{1}{2}a_{e}t^{2}.$$

This means $t^2 = 2L/(a_p - a_e)$ and

$$x = \frac{a_p}{a_p - a_e} L = \frac{eE/m_p}{(eE/m_p) + (eE/m_e)} L = \left(\frac{m_e}{m_e + m_p}\right) L$$
$$= \left(\frac{9.11 \times 10^{-31} \text{kg}}{9.11 \times 10^{-31} \text{kg} + 1.67 \times 10^{-27} \text{kg}}\right) (0.093 \text{ m})$$
$$= 5.1 \times 10^{-5} \text{ m}.$$

42. We place the origin of our coordinate system at point *P* and orient our *y* axis in the direction of the $q_4 = -12e$ charge (passing through the $q_3 = +4e$ charge). The *x* axis is perpendicular to the *y* axis, and thus passes through the identical $q_1 = q_2 = +6e$ charges. The individual magnitudes $|\vec{E}_1|, |\vec{E}_2|, |\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for q_1, q_2 , and q_3 are unnecessary since those charges are positive (assuming q > 0). We note that the contribution from q_1 cancels that of q_2 (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the *y* axis, with magnitude equal to

$$\vec{E}_{\rm net} = \frac{1}{4\pi\varepsilon_0} \left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2} \right) \hat{j} = \frac{1}{4\pi\varepsilon_0} \left(\frac{12e}{4d^2} - \frac{4e}{d^2} \right) \hat{j}.$$

With $d = 5.0 \times 10^{-6}$ m, we find a field magnitude of 57.5 N/C.

58. **THINK** The electric quadrupole is composed of two dipoles, each with a dipole moment of magnitude p = qd. The dipole moments point in the opposite directions and produce fields in the opposite directions at points on the quadrupole axis.

EXPRESS Consider the point *P* on the axis, a distance *z* to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is given by $qd/2\pi\varepsilon_0(z - d/2)^3$ while the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$.

ANALYZE Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

 $(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$

we obtain

$$E = \frac{qd}{2\pi\varepsilon_0 (z - d/2)^3} - \frac{qd}{2\pi\varepsilon_0 (z + d/2)^3} \approx \frac{qd}{2\pi\varepsilon_0} \left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4} \right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}$$

Since the quadrupole moment is $Q = 2qd^2$, we have $E = \frac{3Q}{4\pi\varepsilon_0 z^4}$.

LEARN For a quadrupole moment Q, the electric field varies with z as $E \colon Q/z^4$. For a point charge q, the dependence is $E \colon q/z^2$, and for a dipole p, we have $E \colon p/z^3$.