## Chapter 22

\# 17. First, we need a formula for the field due to the arc. We use the notation $\lambda$ for the charge density, $\lambda=Q / L$. Sample Problem 22.03 - "Electric field of a charged circular rod," illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle $\theta$ ) is

$$
E_{\mathrm{arc}}=\frac{\lambda}{4 \pi \varepsilon_{0} r}[\sin (\theta / 2)-\sin (-\theta / 2)]=\frac{2 \lambda \sin (\theta / 2)}{4 \pi \varepsilon_{0} r} .
$$

Now, the arc length is $L=r \theta$ with $\theta$ expressed in radians. Thus, using $R$ instead of $r$, we obtain

$$
E_{\mathrm{arc}}=\frac{2(Q / L) \sin (\theta / 2)}{4 \pi \varepsilon_{0} R}=\frac{2(Q / R \theta) \sin (\theta / 2)}{4 \pi \varepsilon_{0} R}=\frac{2 Q \sin (\theta / 2)}{4 \pi \varepsilon_{0} R^{2} \theta} .
$$

Thus, the problem requires $E_{\text {arc }}=\frac{1}{2} E_{\text {particle }}$, where $E_{\text {particle }}$ is given by Eq. 22-3. Hence,

$$
\frac{2 Q \sin (\theta / 2)}{4 \pi \varepsilon_{0} R^{2} \theta}=\frac{1}{2} \frac{Q}{4 \pi \varepsilon_{0} R^{2}} \Rightarrow \sin \frac{\theta}{2}=\frac{\theta}{4}
$$

where we note, again, that the angle is in radians. The approximate solution to this equation is $\theta$ $=3.791 \mathrm{rad} \approx 217^{\circ}$.
\# 20. From symmetry, we see that the net field at $P$ is twice the field caused by the upper semicircular charge $+q=\lambda(\pi R)$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$
\vec{E}_{\mathrm{net}}=\left.2(-\stackrel{\rightharpoonup}{\mathrm{j}})^{4 \pi \varepsilon_{0} R} \sin \theta\right|_{-90^{\circ}} ^{90^{\circ}}=-\left(\frac{q}{\varepsilon_{0} \pi^{2} R^{2}}\right) \mathrm{j} .
$$

(a) With $R=4.23 \times 10^{-2} \mathrm{~m}$ and $q=3.90 \times 10^{-11} \mathrm{C},\left|\vec{E}_{\text {net }}\right|=250 \mathrm{~N} / \mathrm{C}$.
(b) The net electric field $\vec{E}_{\text {net }}$ points in the $-\hat{\mathrm{j}}$ direction, or $-90^{\circ}$ counterclockwise from the $+x$ axis.
\# 21. We assume $q>0$. Using the notation $\lambda=q / L$ we note that the (infinitesimal) charge on an element $d x$ of the rod contains charge $d q=\lambda d x$. By symmetry, we conclude that all horizontal field components (due to the $d q$ 's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ( $0 \leq$ $x \leq L / 2$ ) and then simply double the result. In that regard we note that $\sin \theta=R / r$ where $r=\sqrt{x^{2}+R^{2}}$.
(a) Using Eq. 22-3 (with the 2 and $\sin \theta$ factors just discussed) the magnitude is

$$
\begin{aligned}
|\vec{E}| & =2 \int_{0}^{L / 2}\left(\frac{d q}{4 \pi \varepsilon_{0} r^{2}}\right) \sin \theta=\frac{2}{4 \pi \varepsilon_{0}} \int_{0}^{L / 2}\left(\frac{\lambda d x}{x^{2}+R^{2}}\right)\left(\frac{y}{\sqrt{x^{2}+R^{2}}}\right) \\
& =\frac{\lambda R}{2 \pi \varepsilon_{0}} \int_{0}^{L / 2} \frac{d x}{\left(x^{2}+R^{2}\right)^{3 / 2}}=\left.\frac{(q / L) R}{2 \pi \varepsilon_{0}} \cdot \frac{x}{R^{2} \sqrt{x^{2}+R^{2}}}\right|_{0} ^{L / 2} \\
& =\frac{q}{2 \pi \varepsilon_{0} L R} \frac{L / 2}{\sqrt{(L / 2)^{2}+R^{2}}}=\frac{q}{2 \pi \varepsilon_{0} R} \frac{1}{\sqrt{L^{2}+4 R^{2}}}
\end{aligned}
$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item \#19 in the list of integrals). With $q=9.56 \times 10^{-12} \mathrm{C}, L=0.0850 \mathrm{~m}$, and $R=0.0600 \mathrm{~m}$, we have $|\vec{E}|=19.5 \mathrm{~N} / \mathrm{C}$.
(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90^{\circ}$ counterclockwise from the $+x$ axis.
\# 29. The smallest arc is of length $L_{1}=\pi r_{1} / 2=\pi R / 2$; the middle-sized arc has length $L_{2}=\pi r_{2} / 2=\pi(2 R) / 2=\pi R$; and, the largest arc has $L_{3}=\pi(3 R) / 2$. The charge per unit length for each arc is $\lambda=q / L$ where each charge $q$ is specified in the figure. Thus, we find the net electric field to be

$$
E_{\text {net }}=\frac{\lambda_{1}\left(2 \sin 45^{\circ}\right)}{4 \pi \varepsilon_{0} r_{1}}+\frac{\lambda_{2}\left(2 \sin 45^{\circ}\right)}{4 \pi \varepsilon_{0} r_{2}}+\frac{\lambda_{3}\left(2 \sin 45^{\circ}\right)}{4 \pi \varepsilon_{0} r_{3}}=\frac{Q}{\sqrt{2} \pi^{2} \varepsilon_{0} R^{2}}
$$

which yields $E_{\text {net }}=1.46 \times 10^{7} \mathrm{~N} / \mathrm{C}$.
(b) The direction is $-45^{\circ}$, measured counterclockwise from the $+x$ axis.
\# 32. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$
U=-\vec{p} \cdot \vec{E}=-100 \times 10^{-28} \mathrm{~J} .
$$

If $E=40 \mathrm{~N} / \mathrm{C}$, we find $p=2.5 \times 10^{-28} \mathrm{C} \cdot \mathrm{m}$.
\# 33. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_{p}=e E / m_{p}$ and the acceleration of the electron is $a_{e}=-e E / m_{e}$, where $E$ is the magnitude of the electric field, $m_{p}$ is the mass of the proton, and $m_{e}$ is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time $t$ is $x=\frac{1}{2} a_{p} t^{2}$ and the coordinate of the electron is $x=L+\frac{1}{2} a_{e} t^{2}$. They pass each other when their coordinates are the same, or

$$
\frac{1}{2} a_{p} t^{2}=L+\frac{1}{2} a_{e} t^{2}
$$

This means $t^{2}=2 L /\left(a_{p}-a_{e}\right)$ and

$$
\begin{aligned}
x & =\frac{a_{p}}{a_{p}-a_{e}} L=\frac{e E / m_{p}}{\left(e E / m_{p}\right)+\left(e E / m_{e}\right)} L=\left(\frac{m_{e}}{m_{e}+m_{p}}\right) L \\
& =\left(\frac{9.11 \times 10^{-31} \mathrm{~kg}}{9.11 \times 10^{-31} \mathrm{~kg}+1.67 \times 10^{-27} \mathrm{~kg}}\right)(0.093 \mathrm{~m}) \\
& =5.1 \times 10^{-5} \mathrm{~m} .
\end{aligned}
$$

\# 42. We place the origin of our coordinate system at point $P$ and orient our $y$ axis in the direction of the $q_{4}=-12 e$ charge (passing through the $q_{3}=+4 e$ charge). The $x$ axis is perpendicular to the $y$ axis, and thus passes through the identical $q_{1}=q_{2}=+6 e$ charges. The individual magnitudes $\left|\vec{E}_{1}\right|,\left|\vec{E}_{2}\right|,\left|\vec{E}_{3}\right|$, and $\left|\vec{E}_{4}\right|$ are figured from Eq. 22-3, where the absolute value signs for $q_{1}, q_{2}$, and $q_{3}$ are unnecessary since those charges are positive (assuming $q>0$ ). We note that the contribution from $q_{1}$ cancels that of $q_{2}$ (that is, $\left|\vec{E}_{1}\right|=\left|\vec{E}_{2}\right|$ ), and the net field (if there is any) should be along the $y$ axis, with magnitude equal to

$$
\vec{E}_{\mathrm{net}}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{\left|q_{4}\right|}{(2 d)^{2}}-\frac{q_{3}}{d^{2}}\right) \hat{\jmath}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{12 e}{4 d^{2}}-\frac{4 e}{d^{2}}\right) \hat{\jmath}
$$

With $d=5.0 \times 10^{-6} \mathrm{~m}$, we find a field magnitude of $57.5 \mathrm{~N} / \mathrm{C}$.
\# 58. THINK The electric quadrupole is composed of two dipoles, each with a dipole moment of magnitude $p=q d$. The dipole moments point in the opposite directions and produce fields in the opposite directions at points on the quadrupole axis.

EXPRESS Consider the point $P$ on the axis, a distance $z$ to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is given by $q d / 2 \pi \varepsilon_{0}(z-d / 2)^{3}$ while the field produced by the left dipole is $-q d / 2 \pi \varepsilon_{0}(z+$ $d / 2)^{3}$.

ANALYZE Use the binomial expansions

$$
\begin{aligned}
& (z-d / 2)^{-3} \approx z^{-3}-3 z^{-4}(-d / 2) \\
& (z+d / 2)^{-3} \approx z^{-3}-3 z^{-4}(d / 2)
\end{aligned}
$$

we obtain

$$
E=\frac{q d}{2 \pi \varepsilon_{0}(z-d / 2)^{3}}-\frac{q d}{2 \pi \varepsilon_{0}(z+d / 2)^{3}} \approx \frac{q d}{2 \pi \varepsilon_{0}}\left[\frac{1}{z^{3}}+\frac{3 d}{2 z^{4}}-\frac{1}{z^{3}}+\frac{3 d}{2 z^{4}}\right]=\frac{6 q d^{2}}{4 \pi \varepsilon_{0} z^{4}} .
$$

Since the quadrupole moment is $Q=2 q d^{2}$, we have $E=\frac{3 Q}{4 \pi \varepsilon_{0} z^{4}}$.

LEARN For a quadrupole moment $Q$, the electric field varies with $z$ as $E: Q / z^{4}$. For a point charge $q$, the dependence is $E: q / z^{2}$, and for a dipole $p$, we have $E: p / z^{3}$.

