Chapter 31

4. When the switch is open, we have a series *LRC* circuit involving just the one capacitor near the upper right corner. Equation 31-65 leads to

$$\frac{\omega_d L - \dfrac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) = -\tan 20°.$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is $2C$. In this case, we have

$$\frac{\omega_d L - \dfrac{1}{2\omega_d C}}{R} = \tan\phi_1 = \tan 10.0°.$$

Finally, with the switch in position 2, the circuit is simply an *LC* circuit with current amplitude

$$I_2 = \frac{\mathcal{E}_m}{Z_{LC}} = \frac{\mathcal{E}_m}{\sqrt{\left(\omega_d L - \dfrac{1}{\omega_d C}\right)^2}} = \frac{\mathcal{E}_m}{\dfrac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $(\omega_d C)^{-1} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for *L*, *R* and *C* from the three equations above, and the results are as follows:

(a) $R = \dfrac{-\mathcal{E}_m}{I_2 \tan\phi_o} = \dfrac{-120\text{ V}}{(2.00\text{ A})\tan(-20.0°)} = 165\ \Omega$,

(b) $L = \dfrac{\mathcal{E}_m}{\omega_d I_2}\left(1 - 2\dfrac{\tan\phi_1}{\tan\phi_o}\right) = \dfrac{120\text{ V}}{2\pi(60.0\text{ Hz})(2.00\text{ A})}\left(1 - 2\dfrac{\tan 10.0°}{\tan(-20.0°)}\right) = 0.313\text{ H}$,

(c) and

$$C = \frac{I_2}{2\omega_d \mathcal{E}_m\left(1 - \tan\phi_1/\tan\phi_0\right)} = \frac{2.00\text{ A}}{2(2\pi)(60.0\text{ Hz})(120\text{ V})\left(1 - \tan 10.0°/\tan(-20.0°)\right)}$$
$$= 1.49 \times 10^{-5}\text{ F}.$$

6. (a) The rms current is

$$I_{\text{rms}} = \frac{\%_{\text{rms}}}{Z} = \frac{\%_{\text{rms}}}{\sqrt{R^2 + \left(2\pi fL - 1/2\pi fC\right)^2}}$$

$$= \frac{75.0\,\text{V}}{\sqrt{\left(20.0\ \Omega\right)^2 + \left\{2\pi\left(550\,\text{Hz}\right)\left(25.0\ \text{mH}\right) - 1/\left[2\pi\left(550\ \text{Hz}\right)\left(4.70\ \mu\text{F}\right)\right]\right\}^2}}$$

$$= 2.35\,\text{A}.$$

(b) The rms voltage across R is $V_{ab} = I_{\text{rms}} R = \left(2.59\,\text{A}\right)\left(20.0\ \Omega\right) = 47.1\,\text{V}$.

(c) The rms voltage across C is

$$V_{bc} = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{2\pi fC} = \frac{2.35\,\text{A}}{2\pi\left(550\,\text{Hz}\right)\left(4.70\,\mu\text{F}\right)} = 144.85\ \text{V} \approx 145\,\text{V}\ .$$

(d) The rms voltage across L is

$$V_{cd} = I_{\text{rms}} X_L = 2\pi I_{\text{rms}} fL = 2\pi\left(2.35\,\text{A}\right)\left(550\ \text{Hz}\right)\left(25.0\,\text{mH}\right) = 203.25\ \text{V} \approx 203\,\text{V}\ .$$

(e) The rms voltage across C and L together is

$$V_{bd} = \left|V_{bc} - V_{cd}\right| = \left|144.85\,\text{V} - 203.25\,\text{V}\right| = 58.4\ \text{V}\ .$$

(f) The rms voltage across R, C, and L together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{\left(47.1\,\text{V}\right)^2 + \left(58.4\,\text{V}\right)^2} = 75.0\,\text{V}\,.$$

(g) For the resistor R, the power dissipated is $P_R = \dfrac{V_{ab}^2}{R} = \dfrac{\left(47.1\,\text{V}\right)^2}{20.0\,\Omega} = 111\,\text{W}.$

(h) No energy dissipation in C.

(i) No energy dissipation in L.

7. This circuit contains no reactances, so $\mathcal{E}_{\text{rms}} = I_{\text{rms}}R_{\text{total}}$. Using Eq. 31-71, we find the average dissipated power in resistor R is

$$P_R = I_{\text{rms}}^2 R = \left(\frac{\mathcal{E}_m}{r+R}\right)^2 R.$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\mathcal{E}_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\mathcal{E}_m^2 (r-R)}{(r+R)^3} = 0 \quad \Rightarrow \quad R = r$$

18. (a) From Eq. 31-4, we have $L = (\omega^2 C)^{-1} = ((2\pi f)^2 C)^{-1} = 2.41\ \mu\text{H}$.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{\max} = \frac{1}{2}LI^2 = 21.4$ pJ.

(c) Of several methods available to do this part, probably the one most "in the spirit" of this problem (considering the energy that was calculated in part (b)) is to appeal to $U_{\max} = \frac{1}{2}Q^2/C$ (from Chapter 26) to find the maximum charge: $Q = \sqrt{2CU_{\max}} = 82.2$ nC.

42. (a) After the switch is thrown to position *b* the circuit is an *LC* circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(27.0\times10^{-3}\text{ H}\right)\left(6.20\times10^{-6}\text{ F}\right)}} = 389\text{ Hz}.$$

(b) When the switch is thrown, the capacitor is charged to V = 34.0 V and the current is zero. Thus, the maximum charge on the capacitor is

$$Q = VC = (45.0\text{ V})(6.20\times10^{-6}\text{ F}) = 2.79\times10^{-4}\text{ C}.$$

The current amplitude is

$$I = \omega Q = 2\pi fQ = 2\pi\left(275\text{ Hz}\right)\left(2.79\times10^{-4}\text{ C}\right) = 0.682\text{ A}.$$

60. **THINK** We have a series *RLC* circuit. Since *R*, *L*, and *C* are in series, the same current is driven in all three of them.

EXPRESS The capacitive and the inductive reactances can be written as

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C}, \quad X_L = \omega_d L = 2\pi f_d L.$$

The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the current amplitude is given by $I = \varepsilon_m / Z$.

ANALYZE (a) Substituting the values given, we find the capacitive reactance to be

$$X_C = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times 10^{-6}\text{ F})} = 37.894\ \Omega.$$

Similarly, the inductive reactance is

$$X_L = 2\pi f_d L = 2\pi(60.0\text{ Hz})(230\times 10^{-3}\text{ H}) = 86.708\ \Omega.$$

Thus, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(400\ \Omega)^2 + (37.894\ \Omega - 86.708\ \Omega)^2} = 403\ \Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.708\ \Omega - 37.894\ \Omega}{400\ \Omega}\right) = 69.6°.$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\text{ V}}{403\ \Omega} = 89.3\text{ mA}.$$

LEARN The circuit in this problem is more inductive since $X_L > X_C$. The phase angle is positive, so the current lags behind the applied emf.