## Chapter 21

\# 20. Since the graph crosses zero, $q_{1}$ must be positive-valued: $q_{1}=+16 e$. We note that it crosses zero at $r=0.70 \mathrm{~m}$. Now the asymptotic value of the force yields the magnitude and sign of $q_{2}$ :

$$
\frac{q_{1} q_{2}}{4 \pi \varepsilon_{0} r^{2}}=F \Rightarrow q_{2}=\left(\frac{1.5 \times 10^{-25}}{k q_{1}}\right) r^{2}=3.20 \times 10^{-18} \mathrm{C}=20 e .
$$

\# 24. With rightward positive, the net force on $q_{3}$ is

$$
F_{3}=F_{13}+F_{23}=k \frac{q_{1} q_{3}}{\left(L_{12}+L_{23}\right)^{2}}+k \frac{q_{2} q_{3}}{L_{23}^{2}} .
$$

We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on $q_{3}$ by $q_{1}$ ) is negative if they are unlike charges, indicating that $q_{3}$ is being pulled toward $q_{1}$, and it is positive if they are like charges (so $q_{3}$ would be repelled from $q_{1}$ ). Setting the net force equal to zero $L_{23}=$ $2 L_{12}$ and canceling $k, q_{3}$, and $L_{12}$ leads to

$$
\frac{q_{1}}{9.00}+\frac{q_{2}}{4}=0 \Rightarrow \frac{q_{1}}{q_{2}}=-2.25
$$

\# 29. Because the spheres are identical and conducting, each touch of two of them results in their equally sharing their net charge.

Experiment 1: $A+C$ results in each having $+1 Q$. Then $B+C$ results in each having $-2.5 Q$. The magnitude of the force between $A$ and $B$ is

$$
F_{1}=k \frac{(1 Q)(2.5 Q)}{r^{2}}=2.50 k Q^{2} / r^{2}
$$

Experiment 2: $B+C$ results in each having $-3 Q$. Then $A+C$ results in each having $-0.5 Q$. The force between $A$ and $B$ is

$$
F_{2}=k \frac{(0.5 Q)(3.0 Q)}{r^{2}}=1.50 k Q^{2} / r^{2} .
$$

The ratio of the force magnitudes is

$$
\frac{F_{2}}{F_{1}}=\frac{1.5}{2.5}=0.60 .
$$

\# 32. If $\theta$ is the angle between the force and the $x$-axis, then

$$
\cos \theta=\frac{x}{\sqrt{x^{2}+d^{2}}}
$$

We note that, due to the symmetry in the problem, there is no $y$ component to the net force on the third particle. Thus, $F$ represents the magnitude of force exerted by $q_{1}$ or $q_{2}$ on $q_{3}$. Let $e=+1.60$ $\times 10^{-19} \mathrm{C}$, then $q_{1}=q_{2}=+2 e$ and $q_{3}=4.0 e$ and we have

$$
F_{\mathrm{net}}=2 F \cos \theta=\frac{2(2 e)(4 e)}{4 \pi \varepsilon_{0}\left(x^{2}+d^{2}\right)} \frac{x}{\sqrt{x^{2}+d^{2}}}=\frac{4 e^{2} x}{\pi \varepsilon_{0}\left(x^{2}+d^{2}\right)^{3 / 2}} .
$$

(a) To find where the force is at an extremum, we can set the derivative of this expression equal to zero and solve for $x$, but it is good in any case to graph the function for a fuller understanding of its behavior, and as a quick way to see whether an extremum point is a maximum or a miminum. In this way, we find that the value coming from the derivative procedure is a maximum (and will be presented in part (b)) and that the minimum is found at the lower limit of the interval. Thus, the net force is found to be zero at $x=0$, which is the smallest value of the net force in the interval $5.0 \mathrm{~m} \geq x \geq 0$.
(b) The maximum is found to be at $x=d / \sqrt{2}$ or roughly 15 cm .
(c) The value of the net force at $x=0$ is $F_{\text {net }}=0$.
(d) The value of the net force at $x=d / \sqrt{2}$ is $F_{\text {net }}=3.2 \times 10^{-26} \mathrm{~N}$.
\# 41. As a result of the first action, both sphere $W$ and sphere $A$ possess charge $\frac{1}{2} q_{A}$, where $q_{A}$ is the initial charge of sphere $A$. As a result of the second action, sphere $W$ has charge

$$
\frac{1}{2}\left(\frac{q_{A}}{2}-32 e\right)
$$

As a result of the final action, sphere $W$ now has charge equal to

$$
\frac{1}{2}\left[\frac{1}{2}\left(\frac{q_{A}}{2}-32 e\right)+48 e\right]
$$

Setting this final expression equal to $+18 e$ as required by the problem leads (after a couple of algebra steps) to the answer: $q_{A}=+16 e$.

