Chapter 30

1. **THINK** Changing magnetic field induces an electric field.

EXPRESS The induced electric field is given by Eq. 30-20:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}.$$

The electric field lines are circles that are concentric with the cylindrical region. Thus,

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt} \quad \Rightarrow \quad E = -\frac{1}{2}\frac{dB}{dt}r.$$

The force on the electron is $\vec{F} = -e\vec{E}$, so by Newton's second law, the acceleration is

$$\vec{a} = -e\vec{E}/m.$$

ANALYZE (a) At point a,

$$E = -\frac{r}{2}\left(\frac{dB}{dt}\right) = -\frac{1}{2}(5.0\times10^{-2}\,\text{m})(-10\times10^{-3}\,\text{T/s}) = 2.5\times10^{-4}\ \text{V/m}.$$

With the normal taken to be into the page, in the direction of the magnetic field, the positive direction for $\vec{E}$ is clockwise. Thus, the direction of the electric field at point a is to the left, that is $\vec{E} = -(2.5\times10^{-4}\ \text{V/m})\hat{\text{i}}$. The resulting acceleration is

$$\vec{a}_a = \frac{-e\vec{E}}{m} = \frac{(-1.60\times10^{-19}\,\text{C})(-2.5\times10^{-4}\ \text{V/m})}{9.11\times10^{-31}\text{kg}}\hat{\text{i}} = (4.4\times10^{7}\ \text{m/s}^2)\hat{\text{i}}.$$

The acceleration is to the right.

(b) At point b we have $r_b = 0$, so the acceleration is zero.

(c) The electric field at point c has the same magnitude as the field in a, but with its direction reversed. Thus, the acceleration of the electron released at point c is

$$\vec{a}_c = -\vec{a}_a = -(4.4\times10^{7}\ \text{m/s}^2)\hat{\text{i}}.$$

LEARN Inside the cylindrical region, the induced electric field increases with r. Therefore, the greater the value of r, the greater the magnitude of acceleration.

12. (a) Equation 30-8 leads to

$$\% = BLv = (0.480T)(0.300m)(0.55\text{m}/s) = 0.0792V.$$

(b) By Ohm's law, the induced current is

$$i = 0.0792 \text{ V}/18.0\ \Omega = 4.40 \text{ mA}.$$

By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Equation 26-27 leads to $P = i^2R = 0.348$ mW.

15. (a) From Eq. 30-28, we have

$$L = \frac{N\Phi}{i} = \frac{(150)(50\times10^{-9}\ \mathrm{T\cdot m^2})}{2.00\times10^{-3}\ \mathrm{A}} = 3.75\ \mathrm{mH}\,.$$

(b) The answer for L (which should be considered the *constant* of proportionality in Eq. 30-35) does not change; it is still 3.75 mH.

(c) The equations of Chapter 28 display a simple proportionality between magnetic field and the current that creates it. Thus, if the current has doubled, so has the field (and consequently the flux). The answer is 2(50) = 100 nWb.

(d) The magnitude of the induced emf is (from Eq. 30-35)

$$L\left.\frac{di}{dt}\right|_{\max} = (0.00375\ \mathrm{H})(0.0030\ \mathrm{A})(377\ \mathrm{rad/s}) = 4.24\times10^{-3}\ \mathrm{V}\,.$$

37. **THINK** We have an *RL* circuit in which the inductor is in series with the battery.

EXPRESS As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit.

ANALYZE (a) At $t = 0$, the current through the battery is also zero.

(b) With no current anywhere in the circuit at $t = 0$, the loop rule requires the emf of the inductor $\mathcal{E}_L$ to cancel that of the battery ($\mathcal{E} = 40$ V). Thus, the absolute value of Eq. 30-35 yields

$$\frac{di_{\text{bat}}}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{40\ \text{V}}{0.050\ \text{H}} = 8.0\times10^{2}\ \text{A/s}\,.$$

(c) This circuit becomes equivalent to that analyzed in Section 30-9 when we replace the parallel set of 20 kΩ resistors with $R = 10$ kΩ. Now, with $\tau_L = L/R = 5 \times 10^{-6}$ s, we have $t/\tau_L = 3/5$, and we apply Eq. 30-41:

$$i_{\text{bat}} = \frac{\mathcal{E}}{R}\left(1 - e^{-3/5}\right) \approx 1.8\times10^{-3}\,\text{A}.$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 30-35):

$$\mathcal{E} - i_{\text{bat}}R - |\mathcal{E}_L| = 0\,.$$

Using the values from part (c), we obtain $|\mathcal{E}_L| \approx 22$ V. Then,

$$\frac{di_{\text{bat}}}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{22\ \text{V}}{0.050\ \text{H}} \approx 4.4\times10^{2}\ \text{A/s}\,.$$

(e) As $t \to \infty$, the circuit reaches a steady-state condition, so that $di_{\text{bat}}/dt = 0$ and $\mathcal{E}_L = 0$. The loop rule then leads to

$$\mathcal{E} - i_{\text{bat}}R - |\mathcal{E}_L| = 0 \;\Rightarrow\; i_{\text{bat}} = \frac{40\ \text{V}}{10000\ \Omega} = 4.0\times10^{-3}\,\text{A}.$$

(f) As $t \to \infty$, the circuit reaches a steady-state condition, $di_{\text{bat}}/dt = 0$.

LEARN In summary, at $t = 0$ immediately after the switch is closed, the inductor opposes any change in current, and with the inductor and the battery being connected in series, the induced emf in the inductor is equal to the emf of the battery, $\mathcal{E}_L = \mathcal{E}$. A long time later after all the currents have reached their steady-state values, $\%_L = 0$, and the inductor can be treated as an ordinary connecting wire. In this limit, the circuit can be analyzed as if L were not present.

43. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2 B/2$, and the induced emf is

$$\%_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\%_i = \frac{(2.00m)^2}{2}(0.870T/s) = 1.74\text{V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\% + \%_i = 8.00 \text{ V} + 1.74 \text{ V} = 9.74 \text{ V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

63. We use $\varepsilon_2 = -M\, di_1/dt \approx M\, |\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\varepsilon}{\Delta i_1/\Delta t}\right| = \frac{30\times10^3\ \text{V}}{8.0\ \text{A}\big/\left(2.5\times10^{-3}\ \text{s}\right)} = 9.4\ \text{H}\ .$$